① Calculate gcd(356,104)

$$356 = 3\cdot 104 + 44$$
$$104 = 2\cdot 44 + 16$$
$$44 = 2\cdot 16 + 12$$
$$16 = 1\cdot 12 + 4$$
$$12 = 3\cdot 4 + 0$$

$$\boxed{\gcd(356,104) = 4}$$

$$\begin{array}{r} 3 \\ 104\overline{)356} \\ -312 \\ \hline 44 \end{array}$$

$$\begin{array}{r} 2 \\ 44\overline{)104} \\ -88 \\ \hline 16 \end{array}$$

$$\begin{array}{r} 2 \\ 16\overline{)44} \\ -32 \\ \hline 12 \end{array}$$

(2) Yes there are solutions since $\gcd(100,26)=2$ from the calculations given.

We have

$$\boxed{\begin{aligned} 22 &= 100 - 3\cdot 26 \\ 4 &= 26 - 22 \\ 2 &= 22 - 5\cdot 4 \end{aligned}}$$

$$\begin{aligned} 2 &= 22 - 5\cdot 4 \\ &= (100 - 3\cdot 26) - 5\cdot(26-22) \\ &= 100 - 8\cdot 26 + 5\cdot 22 \\ &= 100 - 8\cdot 26 + 5(100 - 3\cdot 26) \\ &= 6\cdot 100 - 23\cdot 26 \end{aligned}$$

$$\boxed{\text{So, } 100x_0 + 26y_0 = 2 \text{ where } x_0 = 6,\ y_0 = -23.}$$

All solutions:

$$\boxed{\begin{aligned} x &= 6 - t\left(\tfrac{26}{2}\right) = 6 - 13t \\ y &= -23 + t\left(\tfrac{100}{2}\right) = -23 + 50t \end{aligned}}$$

3 (a) $\gcd(12,8)=4$

and $4 \nmid 14$

Thus, $12x+8y=14$ has no integer solutions.

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3 (b)

Since $a|b$ we know $ak=b$ for some $k\in\mathbb{Z}$.

Thus, $(ak)^2=b^2$.

So, $a^2(k^2)=b^2$.

Thus, $a^2|b^2$.

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4 A See HW 2 - #9

4 B See HW 1 - #8

5 C

Since $x = \gcd(a,b)$ we know $x \mid a$ and $x \mid b$.
So, $a = xk$, $b = xl$ where $k, l \in \mathbb{Z}$.
Thus, $a+b = x(k+l)$
So $x \mid (a+b)$.
So, x is a positive common divisor of a and $a+b$.
Since $y = \gcd(a, a+b)$, this implies $x \leq y$. $\blacksquare$

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5 D

Assume $\gcd(a,b) = 1$ and $c \mid a$.
Then $a = ck$ where $k \in \mathbb{Z}$.
Let $d = \gcd(b,c)$.
Then, $d > 0$, $d \mid b$, and $d \mid c$.
So, $c = dl$ where $l \in \mathbb{Z}$.
So, $a = ck = dlk$.
So, $d \mid a$.
Since $d \mid a$ and $d \mid b$ and $d > 0$
we know $d = 1$ because $\gcd(a,b) = 1$.
Thus, $\gcd(b,c) = 1$. $\blacksquare$